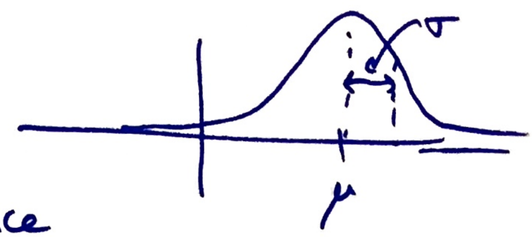
Lecture 28: Continuing normal random variables.

Last time: $\mathcal{N}(\mu, \sigma^2)$

mean, variance



~68% of data within 1 standard deviation of mean μ.

density function:

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Properties:

- total probability is 1. Tricky integral..

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}\, dx = 1$$

(book page 188.)

- $\mathbb{E}[X] = \mu$

$$\mathbb{E}[X] = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} x\, e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}\, dx. \qquad = \mu$$

change of variables
$u = x - \mu$
then another change of vars
$v = u^2$

- Key property:

$$\mathcal{N}(\mu, \sigma^2) = \mu + \sigma\, \mathcal{N}(0,1)$$

So going back to the expectation above

$$\mathbb{E}[\mathcal{N}(\mu,\sigma^2)] = \mu + \sigma\, \mathbb{E}[\mathcal{N}(0,1)]$$

a little easier.

• $\mathrm{Var}[\mathcal{N}(\mu,\sigma^2)] = \sigma^2$ again tricky integral.

$\mathcal{N}(0,1)$ ← standard normal distribution

<u>Def:</u> $\Phi(x)$ = cumulative distribution function of standard normal $\mathcal{N}(0,1)$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$$

because this integral is very hard, there is a table for it in book page 190.

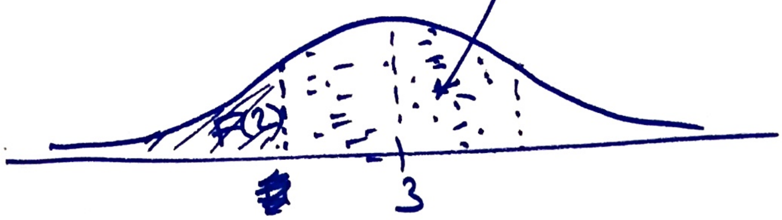
<u>Ex:</u> X normal random variable with mean $\mu = 3$, std. $\sigma = 3$.
find $P(2 < X < 5)$.

<u>Key fact:</u> For X normal random variable.

$$F_X(a) = P(X \leq a) = P\left(\frac{X-\mu}{\sigma} \leq \frac{a-\mu}{\sigma}\right)$$
$$= P\left(\mathcal{N}(0,1) \leq \frac{a-\mu}{\sigma}\right)$$
$$= \Phi\left(\frac{a-\mu}{\sigma}\right)$$

So we can use Φ to get the C.D.F. of any normal random variable.

Solution to problem: $P(\mathcal{N}(3,9) \in [2,5])$



The area we want is between 2 and 5.

$$F(5) - F(2)$$

$$F(5) = P(\mathcal{N}(3,9) \leq 5)$$

$$= P\left(\frac{\mathcal{N}(3,9)-3}{3} \leq \frac{5-3}{3}\right) = P\left(\mathcal{N}(0,1) \leq \frac{2}{3}\right)$$

$$= \Phi(2/3)$$

similarly $F(2) = \Phi(1/3)$.

so $F(5) - F(2) = \Phi(2/3) - \Phi(1/3)$.

Ex: Binary message 0 or 1 is transmitted on a wire from A to B.