Lecture 25:

Continuous random variables:

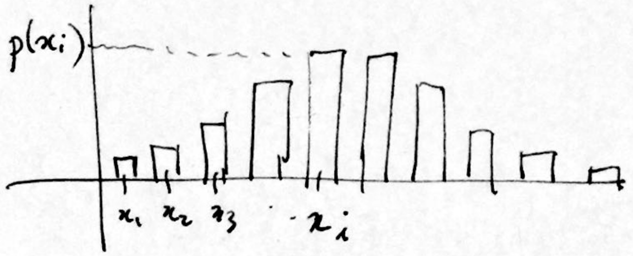
- recall that a random variable X was an object whose value was random <u>in some way</u>.
- discrete random variable could only take finitely or countably infinitely many values

  ie $P(X = x) \neq 0$ for only $x = x_1, x_2, x_3, \ldots$

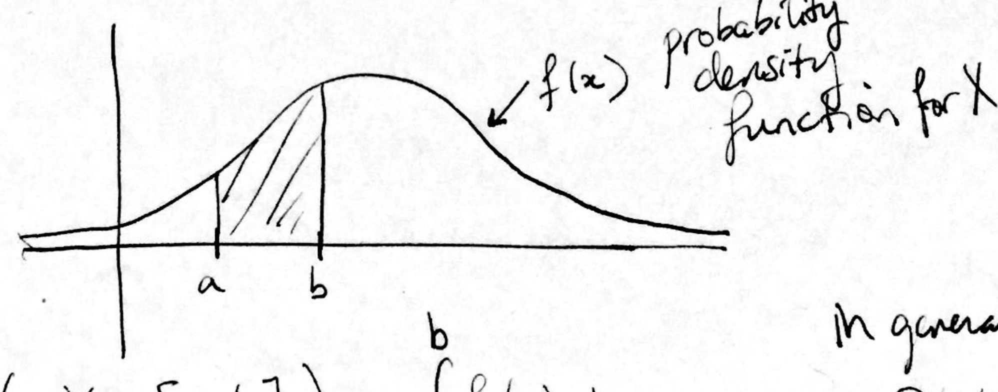
  and $\sum_{i=1}^{\infty} P(X = x_i) = 1$.

- We had probability mass function of X

$$p(x_i) = P(X = x_i)$$



Now we'll do the same for random variables that can take ~~infinitely many~~ values in a continuum.



$$P(X \in [a,b]) = \int_a^b f(x)\,dx$$

In general,

$$P(X \in B) = \int_B f(x)\,dx.$$

of course, we should have:

$$1 = P(X \in (-\infty, \infty)) = \int_{-\infty}^{\infty} f(x)\,dx = 1.$$

and

$$P(X=a) = \int_{a}^{a} f(x)\,dx = 0.$$

so the probability of X taking a specific value is always 0 for a continuous random variable X.

<u>Example</u>: amount of hours that a hard ~~drive~~ drive functions before breaking is a continuous r.v. with probability density function:

$$f(x) = \begin{cases} Ce^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

1) What is C?

2) $P(X \in [50, 150]) = ?$

3) $P(X < 100) = ?$

<u>Solution</u>: We must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

so $$\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} Ce^{-x/100}\,dx = 1$$

$$1 = C\int_{0}^{\infty} e^{-x/100}\,dx = C\cdot(-100)\,e^{-\frac{x}{100}}\Big|_{0}^{\infty} = C\cdot(-100)(0-1)$$
$$= 100C \quad \text{so } C = 1/100$$

2) $P(X \in [50, 150]) = \frac{1}{100}\int_{50}^{150} e^{-\frac{x}{100}}\,dx$

$$= \frac{1}{100}(-100)\, e^{-\frac{x}{100}}\Big|_{50}^{150}$$

$$= e^{-1/2} - e^{-3/2} \approx 0.38$$

3) Similarly,

$$P(X<100) = \int_0^{100} \frac{1}{100} e^{-x/100}\,dx = -e^{-x/100}\Big|_0^{100} = 1-e^{-1} \approx 0.63$$

Definition: the cumulative distribution function of X is defined as:

~~Ex~~:

$$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx.$$

note that f and F can be obtained from each other:

$$\frac{d}{da}F(a) = f(a)$$

(so it's just a different way of expressing the same information)

Question: 1) If X has density function $f(x)$, what is the density function of $Y=2X$?

2) If X has distribution function $F(a)$, what is the distribution function of $Y=2X$?