Lecture 20:

We've been talking about expectation:

$$\mathbb{E}[X] = \sum_{i=1}^{\infty} p(x_i)\, x_i$$

all possible values of X.

<u>Rmk</u>: $\mathbb{E}[aX+bY] = a\,\mathbb{E}[X] + b\,\mathbb{E}[Y]$. expectation is linear!

We noticed that $\mathbb{E}[X^2] \geq \mathbb{E}[X]^2$.

we proved this in class but it's not in the notes.

In fact, this has a name:

$$\mathbb{E}[X^2] - (\mathbb{E}[X])^2 = Var(X)$$

"variance"

but this is not the definition

<u>Def</u>: $\mu = \mathbb{E}[X]$ is called the <u>mean</u> of X

<u>Def</u>: The <u>variance</u> of a random variable X is

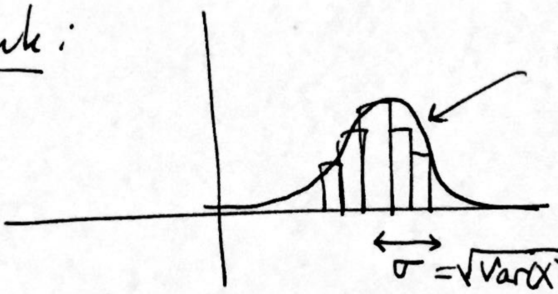
$$\mathbb{E}[(X-\mu)^2]$$

(it measures how 'spread out' the probabilities are)

why is $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$$
\begin{aligned}
Var(X) &= \mathbb{E}[(X-\mu)^2] \\
&= \sum (x_i-\mu)^2 p(x_i) \\
&= \sum (x_i^2 - 2\mu x_i + \mu^2) p(x_i) \\
&= \sum x_i^2 p(x_i) - 2\mu \underbrace{\sum x_i p(x_i)}_{\mu} + \mu^2 \underbrace{\sum p(x_i)}_{1} \\
&= \mathbb{E}[X^2] - 2\mu^2 + \mu^2 = \\
&= \mathbb{E}[X^2] - \mathbb{E}[X]^2
\end{aligned}
$$

Remark:



IF I FIT A 'NORMAL DISTRIBUTION' TO THE PROBABILITY DISTRIBUTION X, then its standard deviation (measure of width) is $\sqrt{Var(X)}$.

(hopefully we'll get to prove this before end of term)

Bernoulli and Binomial random variables:

Bernoulli: X is a Bernoulli random variable

if $P(X=0) = 1-p$
and $P(X=1) = p$ (and therefore these are the only possible values)

e.g.: the outcome of a trial, $0 =$ "failure" ← prob $1-p$.
$1 =$ "success". ← prob p

(exercise $\mathbb{E}[X] = ?$, probability mass function $= ?$)

Binomial: X is a binomial random variable if

it is $\underbrace{Y + Y + \dots + Y}_{n \text{ times}}$ where Y is a Bernoulli random variable.

e.g.: n independent trials are performed, each one is successful with prob. p. $Y = \#$ of successes.

- What is the p.m.f.? n trials with probability p each.

$$p(i) = \binom{n}{i} p^i (1-p)^{n-i}$$