Lecture 19:

Due to popular demand: a quick recap of properties of conditional probability:

We found out before that $P(\cdot|F)$ is a probability. That means that:

1. $P(E|F) \in [0,1]$
2. $P(S|F) = 1$
3. If $E_1, E_2, \ldots$ are mutually exclusive, then $P(\bigcup E_i | F) = \sum P(E_i|F)$.

Since $P(\cdot|E)$ is a probability, everything we know about $P(\cdot)$ applies to $P(\cdot|E)$ as well:

For example:

Each of these has an intuitive idea behind it that will help you use and remember it.

- $P(E_1|F) \geq P(E_2|F)$ if $E_1 \supseteq E_2$.
- $P(E^c|F) = 1 - P(E|F)$
- $P(E_1 \cap E_2|F) \geq P(E_1|F) + P(E_2|F) - 1$
- $P(E|F) = P(EG|F) + P(EG^c|F)$

similarly:

- If $G_1 \cup G_2 \cup \ldots \cup G_n = S$, and $G_1, G_2, \ldots, G_n$ mutually exclusive ($G_i \cap G_j = \phi$), then

$$P(E|F) = P(EG_1|F) + P(EG_2|F) + \ldots + P(EG_n|F)$$

...

We can use these properties in problems as we see fit

Continuing the cool expectation problem:

$$E[X] = \sum_i p(x_i)\, x_i$$

"how much I'm going to get"

probability of getting that.

where $x_1, x_2, \dots$ are all possible values for the discrete random variable X.

<u>Example</u>: In a game show, I can choose the baseball or soccer problem:

Baseball	Soccer
60% chance of getting it right	80%
200\$	100\$

But I can only do the other problem if I get the first problem right. Which should I start with.

<u>solution</u>: ~~Basically~~ we are comparing two strategies, pick 1st problem Baseball or Soccer?

X_1 = "winnings if we pick Baseball first"

X_2 = "winnings if we pick Soccer first".

possible values for X_1 are: 0\$, 200\$, 300\$

probabilities are 0.4, $0.6 \cdot 0.2$, $0.6 \cdot 0.8$

$$E[X_1] = 0.4 \cdot 0\$ + 0.12 \cdot 200\$ + 0.48 \cdot 300\$ = 168\$$$

For X_2: $P(X_2 = 0\$) = 0.2$ $P(X_2 = 300\$) = 0.8 \cdot 0.6 = 0.48$

$P(X_2 = 100\$) = 0.8 \cdot 0.4 = 0.32$

$E[X_2] = \dots \rightarrow$

$\mathbb{E}[X_2] = 144\$ + 32\$ = 176\$ > 168\$$.

Better to start with the easier question
(in exams too!)

<u>Remark/example/question</u>:

Is $\mathbb{E}[X^2] \stackrel{?}{=} (\mathbb{E}[X])^2$

No. Say I win 10\$ if I get H in cointoss
and 0\$ if I get T.

X = my winnings

$\mathbb{E}[X] = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 10 = 5\$$

$\mathbb{E}[X]^2 = 25\$$

$\mathbb{E}[X^2] = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 100 = 50\$$.

In general:

$$\mathbb{E}[g(X)] \neq g(\mathbb{E}[X])$$

$$\mathbb{E}[g(X)] = \sum_{i=1}^{\infty} g(x_i)\, p(x_i)$$

Example: Mad scientist is doing an experiment

wants to make sure the experiment is successful.

$(1-p)=$ probability that experiment is successful.

what is the expected number of times Mad scientisto has to do the experiment to get a success?

solution: $X=$ # of times until experiment is successful.

possible values: $1, 2, 3, \ldots$

$$P(X=i) = p^{i-1}(1-p)$$

$$\mathbb{E}[X] = \sum_{i=1}^{\infty} p^{i}(1-p) \cdot i$$

Example: I have the following options with my 100\$

- Buy stock 1: 0.6 prob of going up to 110
  0.4 " " " down to 90.
- Stock 2: 0.4 prob of going up to 140.
  0.6 " " " down to 80.
- Bank: 1 prob of going up to 101
- Home under mattress: stays 100.

what should I do?

Remark: $\mathbb{E}[X]$ is the mean of many many X's being drawn.