

Lecture 19:

Due to popular demand: a quick recap of properties of conditional probability.

We found out before that $P(\cdot|F)$ is a probability. That means that:

① $P(E|F) \in [0,1]$

② $P(S|F) = 1$

③ If E_1, E_2, \dots are mutually exclusive, then $P(\cup E_i | F) = \sum P(E_i | F)$.

Since $P(\cdot|E)$ is a probability, everything we know about $P(\cdot)$ applies to $P(\cdot|E)$ as well:

For example:

• $P(E_1|F) \geq P(E_2|F)$ if $E_1 \supseteq E_2$.

• $P(E^c|F) = 1 - P(E|F)$

• $P(E_1 \cap E_2 | F) \geq P(E_1|F) + P(E_2|F) - 1$

• $P(E|F) = P(EG|F) + P(EG^c|F)$

similarly:

• If $G_1 \cup G_2 \cup \dots \cup G_n = S$, then $P(E|F) = P(EG_1|F) + P(EG_2|F) + \dots + P(EG_n|F)$ and G_1, G_2, \dots, G_n mutually exclusive. ($G_i \cap G_j = \emptyset$)

We can use these properties in problems as we see fit

Each of these has an intuitive idea behind it that will help you use and remember it.

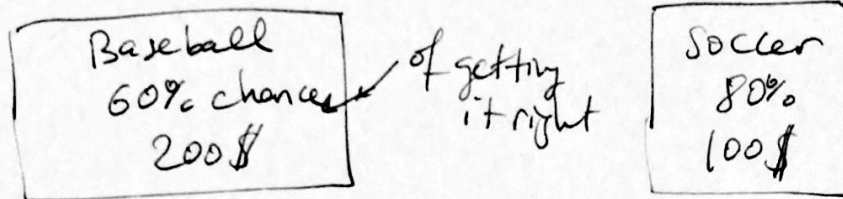
Continuing the cool expectation problem:

$$E[X] = \sum_i p(x_i) x_i$$

"how much I'm going to get"
probability of getting that.

where x_1, x_2, \dots are all possible values for the discrete variable X .
random.

Example: In a game show, I can choose the baseball or soccer problem:



But I can only do the other problem if I get the first problem right. Which should I start with.

solution: Basically we are comparing two strategies, pick 1st problem Baseball or Soccer?

X_1 = "winnings if we pick Baseball first"

X_2 = "winnings if we pick Soccer first"

possible values for X_1 are: 0\$, 200\$, 300\$

probabilities are 0.4, 0.6*0.2, 0.6*0.8

$$E[X_1] = 0.4 \cdot 0\$ + 0.12 \cdot 200\$ + 0.48 \cdot 300\$ = 168\$$$

For X_2 : ~~P(X₂=0\$)=0.2~~ $P(X_2=0\$)=0.2$

$$P(X_2=100\$)=0.8 \cdot 0.4 = 0.32$$

$$P(X_2=300\$)=0.8 \cdot 0.6 = 0.48$$

$$E[X_2] = \dots \rightarrow 2$$

$$E[X_2] = 144\$ + 32\$ = 176\$ > 168\$.$$

Better to start with the easier question
(in exams too!)

Remark/example/question:

$$\text{Is } E[X^2] \stackrel{?}{=} (E[X])^2$$

No. Say I win 10\$ if I get H in coin toss
and 0\$ if I get T.

X = my winnings

$$E[X] = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 10 = 5\$$$

$$E[X]^2 = 25\$$$

$$E[X^2] = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 100 = 50\$.$$

In general:

$$E[g(X)] \neq g(E[X])$$

$$\sum_{i=1}^{\infty} g(x_i) p(x_i)$$

Example: Mad scientist is doing an experiment
wants to make sure the experiment is successful.
 $(1-p)$ = probability that experiment is successful.

What is the expected number of times mad scientist
has to do the experiment to get a success?

Solution: X = # of times until experiment is successful.

possible values: $1, 2, 3, \dots$

$$P(X=i) = p^{i-1}(1-p)$$

$$E[X] = \sum_{i=1}^{\infty} p^{i-1}(1-p) \cdot i$$

Example: I have the following options with my 100\$

- Buy stock 1: 0.6 prob of going up to 110
0.4 " " " down to 90.
- Stock 2: 0.4 prob of going up to 140.
0.6 " " " down to 80.
- Bank: 1 prob of going up to 101
- Home under mattress: stays 100.

what should I do?

Remark: $E[X]$ is the mean of
many many X 's being drawn.