Lecture 18:

Last time: • random variables X

anything related to any probabilistic thing! can be a random variable!

ex: # of heads in n coin tosses $= X$

• discrete random variables.

(Last time I wrote finite but infinite but countable is ok.)

a random var X is discrete if it can take only finitely many values or countably many values.

So there is a list $x_1, x_2, \ldots, x_n$

or $x_1, x_2, x_3, \ldots$

of possible values.

Probability mass function of a discrete random variable X ~~one~~ is a function $p: \{x_1, x_2, \ldots\} \longrightarrow [0,1]$

with $p(x_i) = P(X = x_i)$

Ex: I have two dice but one is fake, it has sides 2,2,3,4,6,6
what is the P.M.F. of $X =$ sum of dice when I roll both.

· possible values: $\{3,4,5,6,7,8,9,10,\cancel{11},12\} \xrightarrow{p} [0,1]$

$p(3) = 1/36$, $p(4) = 1/36, \ldots, p(12) = 3/36$

The upshot of the pmf is that it has all the information I may need about the random variable.

<u>Expectation:</u> If I am taking a bet I have 1/3 chance of winning how much should I request for my win-amount when making the bet?

<u>Ex:</u> my chance of winning is $\frac{1}{3}$. losing is $\frac{2}{3}$.
They are offering 2 dollars if I win
-1 dollars if I lose.

$X =$ "my winnings"

$P(X=2) = \frac{1}{3}$ $\quad P(X=-1) = \frac{2}{3}$

my 'expected' winnings $= \frac{1}{3} \cdot 2 + \frac{2}{3}(-1) = 0$.

so in the long run, it doesn't matter if I keep playing this game. My average winnings will be 0.

<u>Def:</u> $\mathbb{E}[X] = \sum_i x_i \, p(x_i)$

<u>Ex:</u> Expected value of roll of dice $\mathbb{E}[D] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots + 5 \cdot \frac{1}{6} = 3.5$

<u>Ex:</u> In a game show, the contestant can choose amongst two questions 1 and 2. If he answers the first one correctly, he can move on to the second question.

probability of answering 1st correctly is 60% pays 200\$
" 2nd " " 80% " 100\$

Which question should the contestant attempt first?

Solution:

X_1 = winnings if contestant starts with 1st question

X_2 = winnings " " " 2nd question

possible values for X_1:

$0\$$, $200\$$, $300\$$

probabilities: 0.4 , $0.6 \cdot 0.2$, $0.6 \cdot 0.8$

0.4 , 0.12 , 0.48

$$\mathbb{E}[X_1] = 0.4 \cdot 0\$ + 0.12 \cdot 200\$ + 0.48 \cdot 300\$$$

$$= 0 + 24\$ + 144\$ = 168\$$$

For X_2, $p(0\$) = 0.2$ $p(100) = 0.8 \cdot 0.4 = 0.32$ $p(300) = 0.8 \cdot 0.6$

$$\mathbb{E}[X_2] = 0.2 \cdot 0\$ + 0.32 \cdot 100\$ + 0.48 \cdot 300\$$$

$$= 32\$ + 144\$ = 176\$$$

better to start with the easier problems in exams! :)

Ex: ($\mathbb{E}[X^2]$) Recall the game where I toss a coin until I get tails and I win n^2 \$ if I toss n times total.

X = # of times I get to toss the coin

possible values: $1, 2, 3, 4, \ldots$

probabilities $1/2, 1/4, 1/8, \ldots$ $\mathbb{P}(X=n) = \frac{1}{2^n}$

$\mathbb{E}[X]$ $\mathbb{E}[X^2]$

What is the expected # of times I roll? Expected winnings?

$$\mathbb{E}[X] = 1\cdot\frac{1}{2} + 2\cdot\frac{1}{2^2} + 3\cdot\frac{1}{2^3} + \dots = \sum_{i=1}^{\infty} \frac{i}{2^i}$$

$$= \frac{\frac{1}{2}}{(1-\frac{1}{2})^2} = \frac{1/2}{1/4} = 2$$

you don't need to know this

$$\mathbb{E}[X^2] = 1\cdot 1/2 + 4\cdot 1/2^2 + 9\cdot 1/2^3 + \dots = \sum_{i=1}^{\infty} \frac{i^2}{2^i}$$

$$= 6$$

again you don't need to know how to do this.

So $\mathbb{E}[X^2] \neq \mathbb{E}[X]^2$

(the difference is the 'variance')

In general $\mathbb{E}[g(X)] \neq g(\mathbb{E}[X])$

but:

$$\mathbb{E}[g(X)] = \sum_{i=1}^{\infty} g(x_i)\, p(x_i)$$