Lecture 17:

Recap of the course so far:

- counting problems (fun and useful for probability)
- Axioms and properties of probability
- Various cool problems
- Conditional probability, independence, Bayes Theorem and examples.

In order to study expectation and nice continuous distributions, we need to start using the language of:

Random Variables.

Y ← a random variable
← Something that depends on the outcome of a probabilistic event.

Ex: We are tossing a coin 3 times.
$Y =$ number of (H) heads.

"$Y=0$" means no heads
"$Y=1$" means only one head.

$P(Y=0) = 1/8$

$P(Y=1) = 3/8$

$P(Y=2) = 3/8$

$P(Y=3) = 1/8$

observe $\sum_i P(Y=i) = 1$
$\quad\quad\quad\quad \shortparallel$
$P(\bigcup \{Y=i\}) = 1.$

Ex: I won a bet, and as a result I will toss a ~~coin~~ coin until I get T. The number of times I toss the coin will determine how much money I get. If I am able to toss n times, I will get n^2 USD.

$X =$ number of times I get to toss

$Y = X^2$

this is also a random variable (it doesn't have to be directly the event being considered).

$P(Y=5) = 0$ since 5 is not a square

$P(Y \leq 5) = P(Y=1 \text{ or } Y=4)$

$= P(Y=1) + P(Y=4) = P(X=1) + P(X=2)$

$= \underset{\substack{\uparrow \\ \text{coin toss is: T}}}{1/2} + \underset{\substack{\uparrow \\ \text{toss is HT}}}{1/4}$

Ex: GOOG stock price can be considered as a random variable.

Random variables are split into two kinds:

- Discrete Y can only take finite number of different values.
- Continuous Y can be any real number.

• Discrete random variables:

Y takes finitely many different values.

so the outcome is

$Y = a_1$ or $Y = a_2$ or $\cdots$ or $Y = a_n$.

values are among $\{a_1, a_2, \dots, a_n\}$

Or I could say $P(Y = a) = 0$ for $a \notin \{a_1, \dots, a_n\}$

Then we should have:

$$\sum_{i=1}^{n} P(Y = a_i) = 1.$$

You usually write:

$$P(Y = a_i) = p(a_i)$$

• A discrete random variable is determined by values $\{a_1, a_2, \dots, a_n\}$ and a probability $p(a_i) \in [0,1]$ for each a_i.

$$p : \{a_1 \dots a_n\} \longrightarrow [0,1]$$

"probability mass function".

Ex: We toss a coin n times. probability of H is p.
T is $1-p$.

$X = \#$ of H's in n tosses.

what is the probability mass function of X?