Lecture 15:

$P(\cdot|F)$ is a probability.

Recall the axioms of probability.

(1) $1 \geq P(E) \geq 0$

(2) $P(S) = 1$

(3) If $E_1, E_2, \ldots$ mutually exclusive,

then $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$

The same things are true for $P(\cdot|F)$

put E here.

fixed event

(1) $P(E|F) = \frac{P(EF)}{P(F)} \leq 1$ since $P(EF) \leq P(F)$

since $EF \subset F$.

(2) $P(S|F) = \frac{P(SF)}{P(F)} = \frac{P(F)}{P(F)} = 1$

(3) $E_1, E_2, \ldots$ mutually exclusive.

also mutually exclusive $E_i \cap F \cap E_j \cap F = \emptyset$.

$$P(\bigcup E_i|F) = \frac{P((\bigcup E_i) \cap F)}{P(F)} = \frac{P(\bigcup (E_i \cap F))}{P(F)} = \frac{\sum P(E_i F)}{P(F)}$$

$$= \sum P(E_i|F),$$

Example: Back to the example about accident prone people.

A = " person has accident ~~prone~~ ".

R = " person is accident-prone ".

$P(A|R) = 0.4$ $\quad$ $P(R) = 0.3$

$P(A|R^c) = 0.1$

Q: What is the probability that someone who had an accident the 1st year has an accident in the second year.

We need to figure out how likely a person who has an accident is to be accident-prone.

$$P(R|A) = \frac{P(A|R)\,P(R)}{P(A)} = \frac{0.4 \cdot 0.3}{\underbrace{P(A|R)\,P(R) + P(A|R^c)\,P(R^c)}_{0.19}}$$

$$= \frac{12}{19}.$$

then $P(R^c|A) = 1 - P(R|A)$

$$= 7/19.$$

$$P(A_2|A_1) = \underbrace{P(A|R)}_{\text{has accident}}\,\underbrace{P(R|A)}_{\text{accident-prone}} + \underbrace{P(A|R^c)}_{\text{has accident}}\,\underbrace{P(R^c|A)}_{\text{not acc. prone}}$$

$$= 0.4 \cdot \frac{12}{19} + 0.1 \cdot \frac{7}{19}$$

$$= \frac{5.5}{19} \approx 28\%.$$