Lecture 13:

One more Bayes's Theorem example:

Bayes Theorem: $P(E|F) = \dfrac{P(F|E)\,P(E)}{P(F)}$;

we usually write this part as $P(F|E)P(E) + P(F|E^c)P(E^c)$

Example: There are two manufacturers of graphics cards (GPU)

Nvidia and AMD. (made up numbers):

Nvidia makes 80% of cards,	2% of Nvidia cards are defective	
AMD " 20% " "	4% " AMD " " "	

Q: A ~~company~~ person buys a ~~credit~~ graphics! card for their computer and it turns out to be defective. What is the probability that is was Nvidia..

N = "card is Nvidia"

D = "card is defective".

$P(N) = 0.8 \quad P(N^c) = 0.2$

$P(D|N) = 0.02 \quad P(D|N^c) = 0.04$

$$P(N|D) = \frac{P(D|N)\,P(N)}{P(D)} = \frac{0.02 \cdot 0.8}{0.02 \cdot 08 + 0.04 \cdot 0.2} = \frac{16 \times 10^{-3}}{24 \times 10^{-3}} = 2/3$$

$P(D|N)P(N) + P(D|N^c)P(N^c)$

AMD card.

what if there were 3 companies?

		defective rate:
Nvidia:	~~80~~ 70% market share	2%
AMD:	20% " "	4%
Intel:	10% " "	6%.

(say they got into the GPU market)

$$P(N|D) = \frac{\overset{0.02}{P(D|N)} \cdot \overset{0.7}{P(N)}}{P(D)}$$

this one is different

$$P(D) = P(D|N)P(N) + P(D|N^c)P(N^c)$$

not just AMD! could be intel.

$$P(D) = P(D|N)P(N) + P(D|A)P(A) + P(D|I)P(I)$$

A card is either Nvidia or AMD or Intel.
N A I.

these are mutually exclusive events.

Independent events. (different from mutually exclusive)

Definition: Two events E and F are independent if

$$P(EF) = P(E)P(F)$$

this means that ~~the~~ E is equally likely to happen regardless of whether F is assumed or not.

Indeed, $P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(E)P(F)}{P(F)} = P(E)$

ON THE OTHER HAND, IF $P(E|F) = P(E)$ Then $\frac{P(EF)}{P(F)} = P(E)$ SO $P(EF) = P(E)P(F)$

So $P(E|F) = P(E)$

same for $P(F|E) = P(F)$.

So: $P(E) = P(E|F)$ is equivalent to the definition above.

Example: Random card is chosen from deck of 52.

E = "card is spades"

F = "card is ace".

$P(EF) = \frac{1}{52} = \frac{1}{13} \cdot \frac{1}{4} = P(E)P(F)$

Example: I roll two dice.

E = total is 6

F_2 = first die is 2

$P(E, F_2) = P((2,4)) = \frac{1}{36}$

$P(E) = \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$ $P(F_2) = \frac{1}{6}$ $\frac{5}{36} \cdot \frac{1}{6} \neq 1/36$

not independent!

indeed $P(E|F_2) > P(E)$

because E can't happen if first die is 6!

Proposition: E is independent of F if and only if E is independent of F^c.

proof:

$$P(E) = P(EF) + P(EF^c)$$
$$= P(E)P(F) + P(EF^c).$$

assuming $P(E)P(F) = P(EF)$

$$\text{so} \quad P(EF^c) = P(E) - P(E)P(F)$$
$$= P(E)\left(1 - P(F)\right)$$
$$= P(E)\,P(F^c).$$

same proof in other direction. $\square$

For example:

If:
$$P\left(\underbrace{\text{I WILL BE HAPPY FOR THE REST OF MY LIFE}} \,\middle|\, \text{I GET AN A OR BUY A DUCATI MOTORBIKE}\right) = P\left(\underbrace{\text{I WILL BE HAPPY FOR THE REST OF MY LIFE}}\right)$$

then:
$$P\left(\;\middle|\; \text{I DON'T GET AN A AND I DON'T BUY A DUCATI}\right) = P\left(\;^{c}\right).$$

If getting the A doesn't matter, then not getting an A doesn't matter either. Yay!