Lecture 12:

Last time: Bayes' theorem

$$P(E|F) = \frac{P(F|E)\,P(E)}{P(F)}$$

Also the following was discussed:

$$P(F) = P(F|E)P(E) + P(F|E^c)P(E^c).$$

Today: example uses of Bayes' theorem.

Example: Insurance company thinks 0.3 of people are accident-prone, 0.7 not.

If person is accident-prone then 0.4 chance of accident in a year.

If not accident-prone, then 0.1 chance of accident.

If we pick randomly a person who has had an accident, what is the probability that they are accident-prone?

Solution: $P(A)$, $P(R)$, $P(R|A)$, $P(A|R)$

$A =$ "had accident" $R =$ "accident-prone".

$P(R) = 0.3$ $P(A|R) = 0.4$ $P(A|R^c) = 0.1$

$$P(R|A) = \frac{P(A|R)\,P(R)}{P(A)} = \frac{0.4 \cdot 0.3}{P(A)} = \frac{0.12}{0.19} \approx 60\%$$

$$\begin{aligned} P(A) &= P(A|R)P(R) + P(A|R^c)P(R^c) \\ &= 0.4 \cdot 0.3 + 0.1 \cdot 0.7 \\ &= 0.19 \end{aligned}$$

The most famous example:

I go to the doctor and am randomly selected to be tested for disease D.

1% of the population have disease D.

The test has a 1% ~~chance~~ rate of false positive (1% of people who don't have disease D will get positive test results)

Test is 99% accurate for people who have the disease.

If my test result is positive, what is the probability that I have the disease.

D = "I have the disease"

T = "test is positive"

the numbers we have:

$P(D) = 0.01$

$P(T|D) = 0.99$

$P(T|D^c) = 0.01$

$P(D|T) = ?$

THIS IS CALLED OUR 'PRIOR' KNOWLEDGE BEFORE THE TEST.

$$P(D|T) = \frac{P(T|D)\boxed{P(D)}}{P(T)} = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D^c)P(D^c)}$$

$$= \frac{0.99 \cdot 0.01}{0.99 \cdot 0.01 + 0.01 \cdot 0.99} = 0.5 = 50\%$$

Example: Crime and intrigue ...

At a criminal investigation's certain point, inspector is 60% sure that the suspect is guilty.

New evidence comes in: the criminal had blond hair.

Suspect also has blond hair. (guilty!!!)

Assuming 20% of population have blonde hair, what should be the inspector's new belief about the probability of the suspect being (guilty) the criminal.

"a random suspect with 60% chance of being guilty is guilty"

G = "suspect is guilty"

C = "~~suspect~~ a random suspect with 60% chance of being guilty has blond hair."

we know $P(G) = 0.6 \quad P(G^c) = 0.4$

$P(C|G) = 1 \quad P(C|G^c) = 0.2$

$$P(G|C) = \frac{P(C|G)\,\overset{\text{"PRIOR"}}{P(G)}}{P(C)} = \frac{\overset{1}{P(C|G)}\,\overset{0.6}{P(G)}}{\underbrace{P(C|G)}_{1}\underbrace{P(G)}_{0.6} + \underbrace{P(C|G^c)}_{0.2}\underbrace{P(G^c)}_{0.4}}$$

$$= \frac{0.6}{0.6 + 0.08} = \frac{15}{17} \approx 0.88$$

guilt probability now up to 88% from 60%