Lecture 11:

Last time: conditional probability.

$$P(E \mid F) = \frac{P(EF)}{P(F)}$$

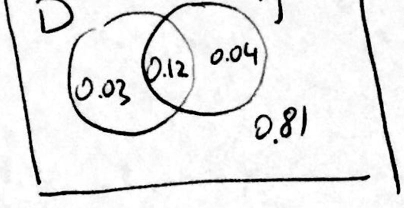
probability of the event E assuming event F happens.

Example: Ingestion of toxin T vs. disease D.

		toxin T - exposure: Yes	toxin T - exposure: No
getting disease D	Yes	0.12	0.03
	No	0.04	0.81

these numbers are the probabilities that a random person will be in each group.

Q: What is $P(\text{I will get disease D} \mid \text{I was exposed to toxin T}) = ?$



$$P(D \mid T) = \frac{P(DT)}{P(T)} = \frac{0.12}{0.16} = 0.75.$$

Lemma: $P(EF) = P(E)\,P(F \mid E)$

probability that E and F both happen.

E happens and F happens assuming E happens.

proof: $P(F \mid E) = \frac{P(EF)}{P(E)}$, multiply both sides with $P(F)$

Ex: I have two dogs, Happy and Sad.

If Happy is barking, there is 0.5 chance that Sad is also barking.

If there is 0.2 chance that Happy is barking then P(they are both barking) = ?

solution: $P(H) = 0.2$

$P(S|H) = 0.5$

$P(SH) = P(H) \cdot P(S|H) = 0.1$.

General version of the multiplication rule:

$$P(E_1 E_2 \dots E_n) = P(E_1)\, P(E_2 | E_1)\, P(E_3 | E_1 E_2) \dots \dots P(E_n | E_1 E_2 \dots E_{n-1}).$$

(proven by repeatedly applying lemma above.)

Bayes' Formula/Theorem:

This is the same thing as the one in the book but the book does this in an unusual way, so keep it in mind when you are studying.

We know:

$$P(EF) = P(E)P(F|E)$$

but also

$$\underset{EF}{\underset{\shortparallel}{P(FE)}} = P(F)P(E|F)$$

so:

$$P(E)P(F|E) = P(F)P(E|F)$$

So

$$\boxed{P(E|F) = \frac{P(F|E)P(E)}{P(F)}}$$

Bayes' theorem.

in practice you usually rewrite $P(F)$ as $P(F) = \underbrace{P(F|E)P(E)}_{P(FE)} + \underbrace{P(F|E^c)P(E^c)}_{P(FE^c)}$

typical example: lab test outcome.

but first.

Example: Insurance company thinks there are two kinds of people: 30% accident-prone and 70% not-accident-prone.

If person is accident-prone then there is 0.4 chance they will have an accident this year. Non-accident-prone: 0.1 ~~0.05~~ chance.

a) What is the probability that a random person will have an accident this year?

b) Say I met someone who just had an accident, what is the probability that they are accident-prone?

Solution: A = "person picked has accident." R = person picked ~~has~~ is accident prone.

a) $$P(A) = P(A|R)\,P(R) + \underbrace{P(A|R^c)}_{\substack{\text{person has accident}\\ \text{assuming not}\\ \text{accident prone}}}\,\underbrace{P(R^c)}_{\substack{\text{person is not}\\ \text{accident prone}}}$$

person is accident prone

and has accident assuming accident prone.

$$P(A) = 0.4 \cdot 0.3 + \cancel{0.7}\; 0.1 \cdot 0.7$$

$$= 0.12 + 0.07 = 0.19.$$

b) We are looking for $P(R|A)$

"accident prone assuming they have an accident".

$$P(R|A) = \frac{P(A|R)P(R)}{P(A)} = \frac{0.4 \cdot 0.3}{0.19} = \frac{0.12}{0.19} = \frac{12}{19}$$

$$\approx 0.6$$

More examples and philosophy next time.