Lecture 10: Conditional probability:

Q: Say I have two coins, but one of them is a Tails-Tails fake coin. The other is a Heads-Tails regular coin. I pick one at random, toss it, got tails, what is the probability that I picked the fair coin?

THIS IS ONE OF THE ASSUMPTIONS OF THE QUESTION.

coin 1: T_1 / T_1'

coin 2 (fair): H_2 / T_2

Sample space when picking at random and tossing:

$S = \{ T_1, T_1', H_2, T_2 \}$ ← uniform probability.

T_1, T_1': 1st coin tails

T_2: second coin tails (fair)

conclusion: $P = 1/3$ because two of the equally likely of the tree outcomes with tails is unfair coin.

Not convinced?. Imagine we had two dice.

one unfair with 6's on all sides
other fair with 1, 2, 3, 4, 5, 6 sides.

I pick one at random, roll, got six!
much more likely that I had picked the second coin.

This kind of thing is called conditional probability.

$P($ I picked fair coin $|$ I got tails when I tossed it $)$ ← "assuming".

Definition: If $P(F) > 0$

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

"probability of E assuming F already happened"

back to coin question: $S = \{T_1, T_1', H_2, T_2\}$

F = "I got tails" $= \{T_1, T_1', T_2\}$

E = "I picked fair coin" $= \{T_2, H_2\}$

$$P(\text{I picked fair coin} \mid \text{I got tails when I tossed it}) = \frac{P(E \cap F)}{P(F)} = \frac{\{T_2\}}{} = \frac{1}{3}.$$

Ex: Prof is grading exams.
Probability that she'll finish in x hours is $\frac{x}{2}$ for $(0 \le x \le 1)$.
Given that she didn't finish after 0.75 hours, what's the probability that she'll still be grading after the full hour?

Solution: events:

F = "grading takes ≥ 1 hour".

L_x = "grading takes $\le x$ hour"

$$P(\underbrace{\text{"didn't finish after 0.75 hours"}}_{L_{0.75}^c}) = 1 - P(L_{0.75})$$

$$P(F \mid L_{0.75}^c) = \frac{P(L_{0.75}^c \cap F)}{P(L_{0.75}^c)} = \frac{P(F)}{1 - P(L_{0.75})} = \frac{0.5}{0.625} = 0.8.$$

(THIS IS ACTUALLY F BECAUSE $F \subset L_{0.75}^c$)