Lecture 5:

Axioms of probability:

Sample space: the set of ~~ever~~ things that can happen.

eg: (1) coin-tossing $S = \{H, T\}$

(2) ~~tossing~~ 3 coins

$$S = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)\}.$$

(3) choosing a random permutation of n elements
$S =$ all $n!$ permutations of n elements

(4) Hard drive failure # of hours

$$S = \{x \in \mathbb{R} \mid x \geq 0\}.$$

Def: An event E is a subset

$$E \subset S$$

eg: (1) coin is H $\quad E = \{H\} \subset S = \{H, T\}$

(2) all coins are same $\quad E = \{(H,H,H), (T,T,T)\}$

or: $E = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T)\}$
first coin is H.

(3) $E =$ subset of S where first element in permutation is 1.

(4) Hard drive fails after 10.000 hours

$$E = (10{,}000, \infty) \subset \mathbb{R}_{>0} = S.$$

We can take union $E \cup F$ or intersection $E \cap F$ $= EF$

either event occurs. both events occur.

this gives a new event. ex: either first coin is heads or all coins are same: $\{(H,H,H),(H,H,T),(H,T,H),(H,T,T)\}$ $\cup \{(H,H,H),(T,T,T)\}$

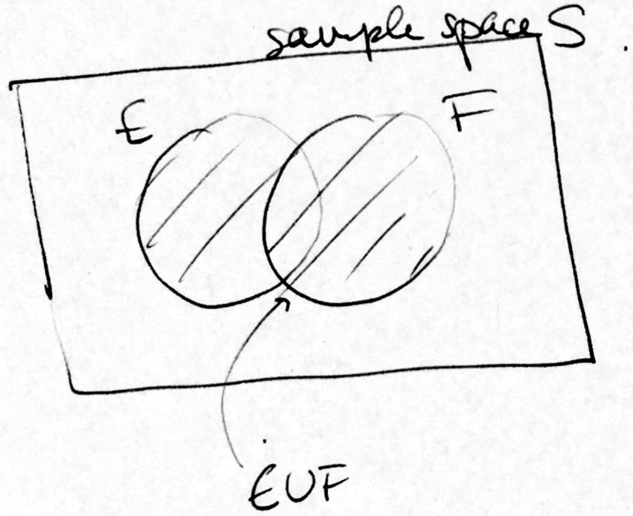
If we have lots of events, we can take:

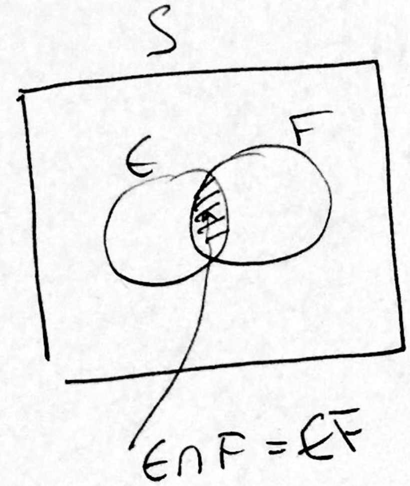
$$E_1 \cup E_2 \cup E_3 \cup \dots = \bigcup_{i=1}^{\infty} E_i$$

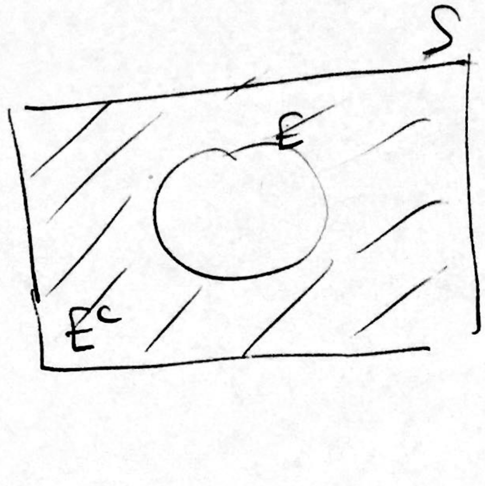
$$E_1 \cap E_2 \cap E_3 \cap \dots = \bigcap_{i=1}^{\infty} E_i$$

We can also take complement. $S \setminus E = E^c = \{x \in S \mid x \notin E\}$ every x in S that is not in E.

<u>diagrams:</u>



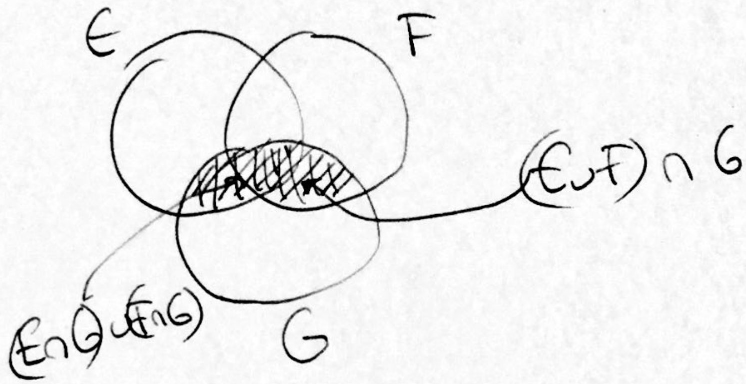




There are algebraic laws:

- $E \cup F = F \cup E$ ✓ commutativity
  $E \cap F = F \cap E$
- associativity: $E \cup (F \cup G) = E \cup (F \cup G)$
- distributivity $(E \cup F) \cap G = (E \cap G) \cup (F \cap G)$

$\cup$ is like $+$ and $\cap$ is like $\cdot$ multiplication.



- Also: "De Morgan's laws".

$$\left(\bigcup_{i=1}^{\infty} E_i\right)^c = \bigcap_{i=1}^{\infty} E_i^c$$

$$\left(\bigcap_{i=1}^{\infty} E_i\right)^c = \bigcup_{i=1}^{\infty} E_i^c.$$

exercise: check with diagram.

- $\phi \subset S$ empty event
  empty set

set with 0 elements

ie something that never happens.

For the foundations of probability, we could consider:

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

$n(E)$ ← # of times event occurred.

n ← total number of times tried.

$P(E)$ ← Probability should be assigned to each event

<u>Other approach</u>: make more self-evident <u>axioms of probability</u> (about $P(E)$) and figure things out from those:

<u>AXIOMS OF PROBABILITY</u>

1. $0 \leq P(E) \leq 1$

2. $P(S) = 1$ ← total probability is always 1.

3. $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

for $E_1, E_2, E_3, \ldots$ <u>independent</u> events.

ie $E_i \cdot E_j = \emptyset$ for all $i \neq j$

(therefore $P(E_i \cdot E_j) = 0$).

(both can't happen together)