Probability Lecture 1:

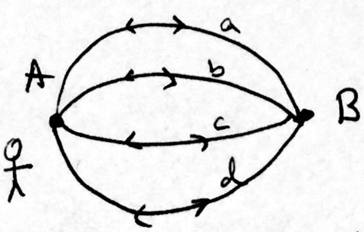
- Course set-up:
  - Homework: 30%
  - Midterm: 20%
  - Problem sessions: 10% (discussion time)
  - Final: 40%.

- Homework policy (collaboration fine but only verbal and you must write everything on your own)

- Why is Probability such a big deal:
  - It allows us to reason with uncertainty (with conclusions also having degrees of uncertainty)
  - It's fun (the problems are fun but don't take my word for it)
  - Useful in physics (quantum physics, data science, machine learning)

§1: Counting

Basic problem:



Person wants to travel to B and back. He picks a random path $A \to B$ and back a random path too. What is the probability that he picked the same path back as he did going forward?

Many ways to look at this, not just one way:

$$p = \frac{\text{\# of back-forth where he choose the same path.}}{\text{\# of all ways of going from A to B and back.}}$$

$$= \frac{|\{aa, bb, cc, dd\}|}{|\{aa, ab, ac, ad, ba, bb, bc, bd, ca, cb, cc, cd, da, db, dc, dd\}|} = \frac{4}{16} = \frac{1}{4}.$$

We could have counted directly:

4 ways to go from A to B. 4 ways to come back from B to A.

total: $4 \cdot 4 = 16$ ways.

Ex: I want to chose two numbers m and n

$m \in \{1, 2, .., M\}$

$n \in \{1, 2, .., N\}$

How many pairs? (m,n) $M \cdot N$

BASIC PRINCIPLE OF COUNTING:

"Suppose two experiments can be performed
experiment 1 can have M ~~outcomes~~ outcomes
experiment 2 can have N outcomes.
Together, there are $M \cdot N$ possible outcomes"

<u>Problem</u>: We have n antennas.

$m \leq n$ of them are defective

They are lined up in a line.

If two consecutive antennas are defective, then system doesn't work.

If the antennas are randomly broken (i.e. broken antennas are randomly placed) what is the probability that the system works?

e.g. $n = 4$
$m = 2$

0 1 1 0 ✓
0 1 0 1 ✓
1 0 1 0 ✓
0 0 1 1 ✗
1 0 0 1 ✗
1 1 0 0 ✗

How do we do this in general?

We need to count.

(we'll come back to this)

Ex: how many functions

$$f: \{1,2,\dots,n\} \to \{0,1\}$$

are there? 2^n

Ex: how many injective functions are there?

$$f: \{1,2,\dots,n\} \to \{1,2,\dots,m\}.$$

- $f(1)$ can be anything (m choices)
- $f(2)$ must be something other than $f(1)$ ($m-1$ choices)

$f(3)$ " " " " " $f(1)$ and $f(2)$:

⋮

answer: $m(m-1)(m-2)\dots(m-n+1)$.

$$= \frac{m!}{(m-n)!}$$

Permutations:

Ex: How many ways can you order $1, 2, \dots, n$?

$\underline{n}\ \underline{n-1}\ \underline{n-2}\ \dots\ \underline{1}$

first place n choices

2nd place $n-1$ choices ...

total: $n(n-1)\dots\cancel{(k)}\dots 3\cdot 2\cdot 1 = n!$