HOMEWORK 7

Due Tuesday, June 6, at the beginning of discussion

1. A student is getting ready to take an important oral examination, and is concerned about the possibility of having an "on" day or an "off" day. He figures that if he has an on day, then each of his examiners will pass him, independently of one another, with probability 0.8, whereas if he has an off day, this probability will be reduced to 0.4. Suppose that the student will pass the exam if a majority of the examiners pass him. If the student believes that he is twice as likely to have an off day as he is to have an on day, should he request an examination with 3 examiners or 5 examiners?

2. The monthly worldwide average number of worldwide celebrity hacking incidents is 4. The editors of a paparizzi magazine are trying to figure out whether they can take time off this month. They need to know: What is the probability that there will be
   (a) at least 3 such celebrity hacking incident in the next month?
   (b) at most 1 celebrity hacking in the next month?
   Explain your reasoning.

3. A certain typing agency employs 2 typists. The average number of errors per article is 3 when typed by the first typist and 4.2 when typed by the second. If your article is equally likely to be typed by either typist, what is the probability that it will have no errors.

4. Independent trials, each resulting in success with probability p and failure with probability $1 - p$, are performed until a trial results in success. We know that the random variable, X, denoting the number of trials required to get the first success, is Geometric with parameter p. Suppose Y is the random variable that counts the number of trials resulting in failure before the first success.
   (a) What are the possible values that Y can take?
   (b) What is the probability mass function of Y?
   (c) Compute $E[Y]$ from what you know about $E[X]$.

5. If X is a geometric random variable, show analytically that

$$P\{X = n + k | X > n\} = P\{X = k\}.$$

Using the interpretation of a geometric random variable, give a verbal argument as to why the preceding equation is true.

6. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10, \\ 0 & x \leq 10. \end{cases}$$

   (a) Find $P\{X > 20\}$.
   (b) What is the cumulative distribution function of X?
   (c) Find $E[X]$.

7. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x}, x \geq 0.$$

   Compute the expected lifetime of such a tube.

8. A point is chosen at random on a line segment of length L. Interpret this statement, and find the probability that the ratio of the shorter to the longer segment is less than $\frac{1}{4}$.

9. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. (Honestly, this is how public transport at Irvine works!)
   (a) What is the probability that you will have to wait longer than 10 minutes?
   (b) If, at 10:15, the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

10. Suppose that the height, in inches, of a 25-year-old man is a normal random variable with parameters $\mu = 71$ and $\sigma^2 = 6.25$. What percentage of 25-year old men are over 6 feet, 2 inches tall? What percentage of men in the 6-footer club are over 6 feet, 5 inches?