HOMEWORK 6

Due Thursday, May 25, at the beginning of discussion

1. Suppose that two teams play a series of games that ends when one of the teams has won k games. Say each game played is, independently, won by team A with probability p. Find the expected number of games that are played when (a) $k = 2$ and (b) $k = 3$. Also show that in both cases this number is maximized when $p = \frac{1}{2}$.

2. Suppose you have two candidates in a presidential election. There are 11 states, and in each state, candidate A wins with probability 0.6 and B wins with probability 0.4. The candidate who wins 6 of the 11 states wins. What is the probability that A wins the election.

3. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimated that E will occur within a year with probability p, what should it charge the customer in order that its expected profit will be 10 percent of A?

4. Suppose three coins are to be flipped. The first coin will land heads with probability 0.6, the second will land heads with probability 0.7 and the third will land heads with probability 0.8. Assume that the results of the flips are independent. Let X denote the total number of heads when flipping all three coins.
   (a) TRUE of FALSE: X is a Binomial random variable? (In either case, explain your answer.)
   (b) What are the possible values that X can take?
   (c) Find the probability mass function of X.
   (d) Find $E[X]$.

5. If X is a random variable with $E[X] = 1$ and $Var(X) = 5$, find
   (a) $E[X^2]$
   (b) $E[(3 - X)^2]$
   (c) $Var(6X - 13)$.

6. Suppose that it takes at least 9 votes from a 12-member jury to convict a defendant (not how it works in real life). Suppose also that the probability that a juror votes a guilty

person innocent is 0.2, whereas the probability that the juror votes an innocent person guilty is 0.1. If each juror acts independently and if 65% of the defendants are guilty,

(a) find the probability that the jury renders a correct decision.

(b) What percentage of the defendants are convicted?

7. A man claims to have extrasensory perception. As a test, a fair coin is flipped 10 times and the man is asked to predict the outcome in advance. He gets 7 out of 10 correct. What is the probability that he would have done at least this well if he did not have extrasensory perception?

8. Weird random variables: construct a discrete random variable X (i.e. describe its probability mass function) such that

   (a) $E[X] = -\infty$.

   (b) $E[X]$ does not exist.

   (c) $E[X]$ exists, is a finite number, but $Var(X) = \infty$.

9. Suppose the average number of lions seen on a 1-day safari is 5. Use the Poisson distribution to find the probability that tourists will see fewer than four lions on the next 1-day safari?