HOMEWORK 5

Due Thursday, May 18, at the beginning of discussion

1. There is a $50-50$ chance that the queen carries the gene for hemophilia. If she is a carrier, then each prince has a $50-50$ chance of having hemophilia. If the queen has had three princes without the disease, what is the probability that a fourth prince, will have hemophilia? (there are no princesses in this country for some reason)

2. Independent flips of a coin that lands on heads with probability p are made. What is the probability that the first four outcomes are
   (a) H,H,H,H?
   (b) T,H,H,H?
   (c) What is the probability that the pattern T, H, H, H occurs before the pattern H, H, H, H? *Hint: How can the pattern H, H, H, H occur first?*

3. A true-false question is to be posed to a husband-and-wife team on a quiz show. Both the husband and the wife will independently give the correct answer with probability p. Which of the following is a better strategy for the couple?
   (a) Choose one of them and let that person answer the question.
   (b) Have them both consider the question, and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.

4. Suppose that n independent trials, each of which results in any one of the outcomes $0, 1$, or 2, with respective probabilities p_0, p_1, and p_2, $\sum_{i=0}^{2} p_i = 1$, are performed. Find the probability that outcomes 1 and 2 both occur at least once.

5. In class, we learnt that if F is an event with $P(F) > 0$, then $P(\cdot|F)$ is a probability - that is, it satisfies the three axioms of probability. Now, will $P(F|\cdot)$ also be a probability? If you think yes, show that it satisfies all the three axioms of probability, and if you think no, give a counter example where one probability axiom or property fails to be satisfied.

6. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. That is,

$$X = |\#(Heads) - \#(Tails)|\,.$$

   (a) What are the possible values of X?

(b) For $n = 4$, write down the probability mass function of X.

7. Let X be the winnings of a gambler. Let $p(i) = P(X = i)$ and suppose that

$$p(0) = 1/3,$$
$$p(1) = p(-1) = 5/36,$$
$$p(2) = p(-2) = 1/6,$$
$$p(3) = p(-3) = 1/36.$$

Compute the conditional probability that the gambler wins $i, i = 1, 2, 3$, given that he wins a positive amount.

8. Roulette! A gambling book recommends the following "winning strategy" for the game of roulette: Bet \$1 on red. If red appears, then take the \$1 profit and quit. If red does not appear and you lose this bet, make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote your winnings when you quit.
   (a) Find $P(X > 0)$.
   (b) Do you think that this strategy is indeed a "winning strategy"? Explain why or why not.
   (c) Find $E[X]$.

   Note 1: Keep in mind that X may be negative.
   Note 2: If you're never seen a roulette wheel before, here's the wiki page (check only American Roulette):
   http://en.wikipedia.org/wiki/Roulette.
   There are 38 total slots for the little ball: 18 red, 18 blue, 1 marked 0 and 1 marked 00.