HOMEWORK 4

Due Thursday, May 4, at the beginning of discussion

1. A district has, amongst eligible voters, (self-declared): 46 percent independents, 30 percent liberals, and 24 percent conservatives. According to the exit polls, 35 percent of the independents, 58 percent of the liberals, and 65 percent of the conservatives voted.
   (a) What percentage of eligible voters voted? (i.e. what was the turnout?)
   (b) If a voter is picked at random, given that they actually voted, what is the probability that they are independent, liberal, conservative?
2. An urn initially contains 6 white and 8 black balls. At each step, a ball is selected at random, its color noted, and it is then replaced in the urn along with two other balls of the same color. Compute the probability that
   (a) the first 2 balls selected are black, and the next 2 are white.
   (b) of the first 4 balls selected, exactly 2 are black.

3. Suppose that an ordinary deck of 52 cards is shuffled, and the cards are then turned over one at a time until the first ace appears. If the first ace is the 20th card, what is the probability that the next card turned is
   (a) ace of spades?
   (b) two of clubs?

4. A family has j children with probability p_j, where $p_1 = 0.1$, $p_2 = 0.25$, $p_3 = 0.35$, $p_4 = 0.3$. A child from this family is randomly chosen. Given that this child is the eldest child in the family, find the probability that the family has
   (a) only 1 child.
   (b) 4 children.
   Redo (a) and (b) when the randomly selected child is the youngest in the family.

5. Barbara and Diane are shooting wooden ducks at the county fair. Suppose that each of Barbara's shots hits a wooden duck target with probability p_1, while each of Diane's hits it with probability p_2. Suppose that they shoot simultaneously at the same target, and the wooden duck gets hit. What is the probability that
   (a) both shots hit the duck?
   (b) Barbara's shot hit the duck?

What independence assumptions have you made?

6. Suppose that E and F are two non-null events. Is it possible that E and F are both independent and mutually exclusive? Explain why or why not.

7. Suppose that E and F are mutually exclusive events of an experiment. Show that if independent trials of this experiment are performed, then E will occur before F with probability
$$\frac{P(E)}{P(E)+P(F)}.$$

8. You go to see the doctor about your hair growing too fast. The doctor informs you that you are not turning into a wolverine, but selects you at random to have a blood test for the K-flu, which for the purposes of this exercise we will say is currently suspected to affect 1 in 10,000 people in Irvine. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is 0.01%. You test positive. What is the new probability that you have K-flu?

   Now imagine that you went to a friends wedding in Imaginia, and (for the purposes of this exercise) it is know that 1 in 200 people who visited Imaginia recently come back with swine flu. Given the same test result as above, what should your revised estimate be for the probability you have the disease?

9. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 9, or when B rolls the sum 6. Assuming that A rolls first, find the probability that the final roll is made by A.