HOMEWORK 2

Due Thursday, April 20, at the beginning of discussion

1. Prove the following identity
$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}.$$
Hint: You can use a combinatorial proof that is in some ways similar to the proof we did in class of the identity $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$.

2. A box contains 3 marbles: 1 red, 1 green and 1 yellow. Consider an experiment that consists of randomly taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box.
   (a) Describe the sample space in words and in set form.
   (b) Suppose the second marble is drawn without replacing the first marble. Again, describe the sample space in words and in set form.
   (c) In the case of (b), describe, in words and in set form, the event that the green ball escaped being drawn.

3. Two dice are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe, both in sentences and in set-notation, the events
   (a) $E \cap F$,
   (b) $E \cup F$,
   (c) $F \cap G$,
   (d) $E \cap F^c$,
   (e) $(E \cap F \cap G)^c$.

4. A, B, and C take turns flipping a coin (assume that A flips first, then B, then C, then A, and so on). The first one to get a head wins and the experiment stops. The sample space of this experiment can be defined by
$$S = \{1, 01, 001, 0001, \ldots\} \cup \{0000\ldots\}.$$
   (a) Interpret, in words, the sample space.
   (b) Define the following events in terms of S:

(i) E_A = the event that A wins.
(ii) E_B = the event that B wins.
(iii) $(E_A \cup E_B)^c$.

5. Let E and F be two events. Let G be the event when exeactly one of E or F happens.
   (a) Write G in set-form.
   (b) Prove (usnig the axioms of probability) that $P(G) = P(E) + P(F) - 2P(EF)$.
6. Prove (using the axioms of probability) that

$$P(EF^c) = P(E) - P(EF).$$

   Additionally, draw a Venn diagram that explains the situation.
7. Suppose that A and B are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that
   (a) either A or B occurs?
   (b) A occurs but B does not?
   (c) both A and B occur?

8. If it is assumed that all $\binom{52}{5}$ 5-card poker hands are equally likely, what is the probability of being dealt
   (a) a flush? *(all 5 cards in the hand are of the same suit)*
   (b) one pair? *(there are 2 cards of the same number, say, a, and the remaining 3 cards have numbers b, c, and d, where a, b, c, and d are all distinct)*
   (c) two pairs? *(there are 2 cards of the same number, say, a, 2 other cards of the same number, b, and the remaining card has number c, where a, b, and c are all distinct)*
   (d) three of a kind? *(there are 3 cards of the same number, say, a, and the remaining 2 cards have numbers b and c, where a, b, and c are all distinct)*
   (e) four of a kind? *(there are 4 cards of the same number, say, a, and the remaining card has number b, where a and b are distinct)*