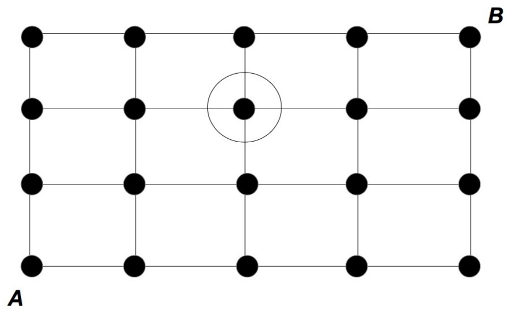
HOMEWORK 1

Due Thursday, April 13, at the beginning of discussion

1. Alice, Bob, Charlie and Dobo have formed a musical band consisting of 4 instruments.
   (a) If each of the boys can play all 4 musical instruments, how many different arrangements are possible?
   (b) What if Alice and Bob can play all 4 instruments, but Charlie and Dobo can each play only piano and drums?
2. In how many ways can 8 people be seated in a row if
   (a) there are no restrictions on the seating arrangement?
   (b) persons A and B must sit next to each other?
   (c) there are 4 men and 4 women and **no 2 men or 2 women** can sit next to each other?
   (d) there are 5 men and they must sit next to one another?
   (e) there are 4 married couples, and each couple must sit together?
3. In how many ways can 3 novels, 2 mathematics books, and 1 chemistry book be arranged on a bookshelf if
   (a) the books can be arranged in any order?
   (b) the mathematics books must be together and the novels must be together?
   (c) the novels must be together, but the other books can be arranged in any order?
4. In a group meeting of 20 people, if everyone shakes hands with everyone else at the start of the meeting, how many handshakes take place?
5. Five separate awards are to be presented to selected students from a class of 30. How many different outcomes are possible if
   (a) a student can receive any number of awards?
   (b) each student can receive at most 1 award?
6. Consider the following grid of points. Suppose that, starting at the point labeled A, you can go one step up or one step to the right at each move. This procedure is continued until the point labeled B is reached.
   (a) How many different paths are there from A to B that go through the point circled in the following lattice?
   (b) How many different paths are there from A to B that **do not** go through the point circled in the following lattice? *Remark: Do not just count the paths visually. Give a mathematical/combinatorial explanation for your answers.*



(c) Now, say we are in a general $n \times n$ grid. We are at the bottom-left at $(0, 0)$, and, at each step, we can travel either one unit above or one unit to the right. Say I have a special point (a, b). Write down a formula for the number of ways we can travel from the bottom-left $(0, 0)$ to the top-right (n, n) by going through (a, b).

7. How many vectors $(x_1, x_2, ..., x_k)$ are there, for which, each x_i is a positive integer with $1 \leq x_i \leq n$ and $x_1 < x_2 < ... < x_k$?

8. Say I have n points on the plane. If I take two of these points, there is, of course, a unique line that goes through the two. I know that only three of these points are on the same line. Other than those three, no other three points are on the same line. How many distinct lines go through these points?

9. Make a combinatorial argument for proving

$$\binom{n}{k} = \sum_{i=k}^{n} \binom{i-1}{k-1}.$$

Idea: Consider the numbers 1 through n, how many subsets of size k have i as the highest numbered member?