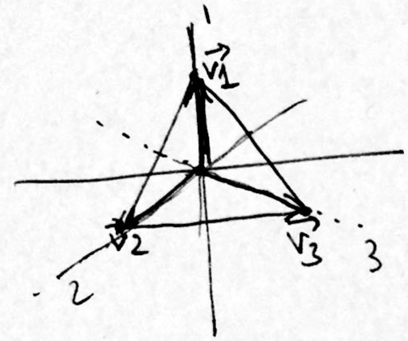
Dihedral group:

Last time, we talked about $GL_n(\mathbb{R}) = \left(\begin{Bmatrix}\text{invertible} \\ n \times n \\ \text{matrices}\end{Bmatrix}, \cdot\right)$

We will now talk about a subgroup $D_6 \subset GL_2(\mathbb{R})$.

$D_6 =$ " {2x2 matrices which will keep a equilateral triangle centered at the origin the same".



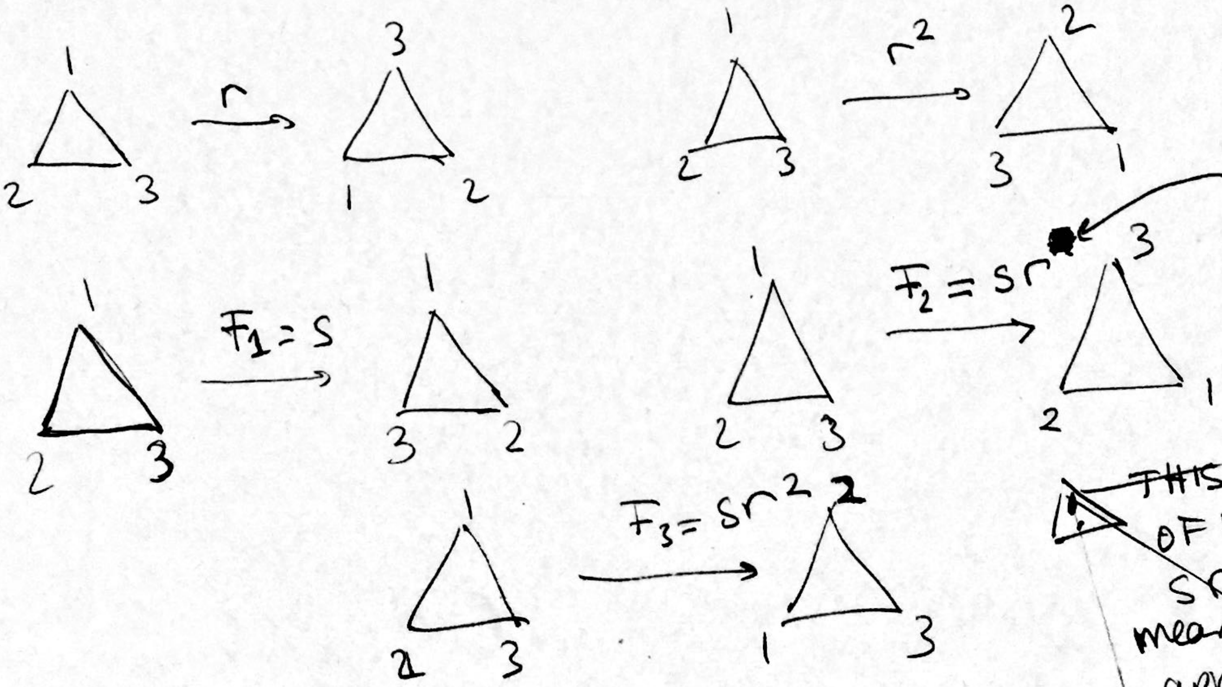
without proof, we are noticing that rotations by 120° and 240°, and flipping along 1, 2, 3 lines will keep triangle the same.

rotation matrix

$I, \; R_{2\pi/3}, \; R_{4\pi/3}, \; F_1, \; F_2, \; F_3$ we are too lazy to write the matrix for these.

$R_{2\pi/3} = r$, $R_{4\pi/3} = r^2$, $F_1 = s = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

easier to see: (these matrices are determined by where they take v_1, v_2 and v_3)



first rotate then flip.

FIXED IT!

~~THIS WAY OF WRITING sr means first apply s then r.~~ ~~opposite order of matrix multiplication~~

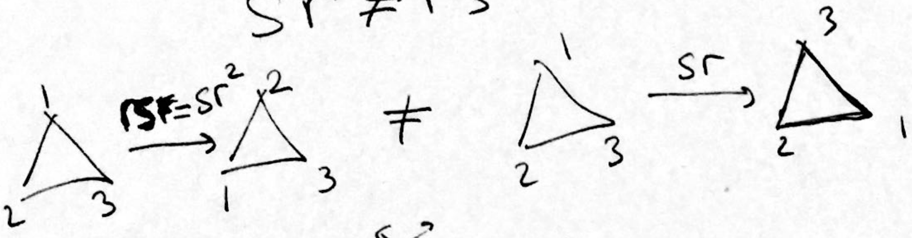
So $D_6 = \{1, r, r^2, s, sr, sr^2\}$

also observe: $r^3 = 1, \; s^2 = 1, \; srs = r^{-1} \; (= r^2)$

this info is actually enough to know everything about the group D_6.

<u>remark</u>: D_6 is not abelian:

$$sr \neq rs$$



not same!

$$D_8 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}.$$
$$\text{with } r^4 = 1, \quad s^2 = 1, \quad srs = r^{-1} = r^3.$$

$$D_{2n} = \{1, r, r^2, \ldots, r^{n-1}, s, sr, \ldots, sr^{n-1}\}$$
$$r^n = 1, \quad s^2 = 1, \quad srs = r^{-1} \; (= r^{n-1})$$

So you write: "presentation of the group".

$$D_{2n} = \Big\langle \underbrace{r, s}_{\text{generators}} \;\Big|\; \underbrace{\begin{matrix} r^n = 1, & srs = r^{-1} \\ s^2 = 1 & \end{matrix}}_{\text{relations}} \Big\rangle$$