HOMEWORK 6

Due Thursday, May 25, at the beginning of discussion

1. Is the map $\mathbb{Z} \times \mathbb{Z} \to S_3$ given by $(i, j) \mapsto (12)^i(123)^j$ a homomorphism? Prove your answer.
2. Find all possible orders in A_5. (Do not list all the elements, just think of all the possible structures of a permutation in A_5 as a product of disjoint cycles).
3. Prove, using induction on n, that every element in S_n is a product of transpositions.
4. Prove that every permutation in A_n can be written as a product of 3-cycles.
5. Suppose G is a group, $a \in G$ and H is a subgroup of G.
   (a) Show that if $aH = H$ then $a \in H$.
   (b) Show that if $a \in H$, then $aH = H$. (Hint: Assume a belongs to H. Then prove double inclusion: aH is a subset of H, and viceversa.)
   (Together, they imply that $aH = H$ if and only if $a \in H$.)
6. Suppose G is a group, $a \in G$ and H is a subgroup of G.
   (a) Show that aH and H have the same cardinality by exhibiting a bijection between the two sets. (Similarly, one can show that Ha and H have the same cardinality, so aH and Ha always have the same cardinality.)
   (b) Show, by means of an example, that aH is not necessarily equal to Ha. (Hint: it has to be a non-commutative group)
7. Suppose G is a group, $a \in G$ and H is a subgroup of G. Show that $aH = Ha$ if and only if $aHa^{-1} = H$.
8. (a) Recall that we can view D_{2n} as a subgroup of S_n. Find the partition of S_4 into left cosets of D_8.
   (b) Let $H = SL_2(\mathbb{R})$ denote the subgroup of $GL(2, \mathbb{R})$ consisting of all matrices with determinant 1. Describe the partition of $GL(2, \mathbb{R})$ into left cosets of H.