MATH 3A: HOMEWORK 5

Due Tuesday, Nov 8st, at the beginning of your discussion session

1. Let $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. Construct a 4×2 matrix D using only 1 and 0 as entries, such that $AD = I_2$. Is it possible that $CA = I_4$ for some 4×2 matrix C? Why or why not?
2. Figure out if these matrices are invertible. You can use any method. You don't need to find the actual inverse for each invertible one.

$$\begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 5 & 9 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix}$$

3. An upper-triangular matrix is a matrix whose entries below the diagonal (i.e. when $i < j$) are zero. When is an upper-triangular matrix invertible? Justify your answer.
4. Can a square matrix with two identical columns be invertible? Why or why not?
5. Is it possible for a 5×5 matrix to be invertible when its columns do not span $\mathbb{R}^5$? Why or why not?
6. If C is 6×6 and the equation $C\mathbf{x} = \mathbf{v}$ is consistent for every $\mathbf{v}$ in $\mathbb{R}^6$, is it possible that for some $\mathbf{v}$, the equation $C\mathbf{x} = \mathbf{v}$ has more than one solution? Why or why not?
7. If the equation $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ in $\mathbb{R}^n$, what can you say about the equation $H\mathbf{x} = \mathbf{0}$? Why?
8. Explain why the columns of A^2 span $\mathbb{R}^n$ whenever the columns of A are linearly independent.
9. Show that if AB is invertible, so is A. You cannot use Theorem 6(b), because you cannot *assume* that A and B are invertible. [*Hint:* There is a matrix W such that $ABW = I$. Why?]
10. Given $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, find a vector in Nul A and a vector in Col A.
11. Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ -5 \\ 8 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 8 \\ 2 \\ -9 \end{bmatrix}$. Determine if $\mathbf{w}$ is in the subspace of $\mathbb{R}^3$ generated by $\mathbf{v}_1$ and $\mathbf{v}_2$.
12. For the following matrix A, find p and q so that $\text{Nul}\,A \subset \mathbb{R}^p$ and $\text{Col}\,A \subset \mathbb{R}^q$. Then find a non-zero vector in each of these subspaces

$$A = \begin{bmatrix} 3 & 2 & 1 & -5 \\ -9 & -4 & 1 & 7 \\ 9 & 2 & -5 & 1 \end{bmatrix}$$

13. Find out whether each of the following sets of vectors is a basis of $\mathbb{R}^2$ or $\mathbb{R}^3$ (each line's a different set)

$$\begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 10 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -7 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -8 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -6 \\ -7 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 9 \end{bmatrix}$$

14. For the following matrices whose echelon forms are given, find a basis of the null spaces and the column spaces. What is the rank and nullity of each one? What is rank

+ nullity?

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

15. Construct a nonzero 3×3 matrix A and a vector $\mathbf{b}$ such that $\mathbf{b}$ is *not* in Col A.

16. Construct a nonzero 3×3 matrix A and a nonzero vector $\mathbf{b}$ such that $\mathbf{b}$ is in Nul A.

17. True/False? Please justify your answer.

    A subspace of $\mathbb{R}^n$ is any set H such that (i) the zero vector is in H, (ii) $\mathbf{u}, \mathbf{v}$, and $\mathbf{u} + \mathbf{v}$ are in H, and (iii) c is a scalar and $c\mathbf{u}$ is in H.

    If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are in $\mathbb{R}^n$, then Span $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is the same as the column space of the matrix $[\mathbf{v}_1 \ \cdots \ \mathbf{v}_p]$.

    The set of all solutions of a system of m homogeneous equations in n unknowns is a subspace of $\mathbb{R}^m$.

    The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

    A subset H of $\mathbb{R}^n$ is a subspace if the zero vector is in H.

    Given vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$, the set of all linear combinations of these vectors is a subspace of $\mathbb{R}^n$.

    The null space of an $m \times n$ matrix is a subspace of $\mathbb{R}^n$.

18. What can you say about Nul B when B is a 5×4 matrix with linearly independent columns?

19. What is the rank of a 4×5 matrix whose null space is three-dimensional?

20. If the subspace of all solutions of $A\mathbf{x} = \mathbf{0}$ has a basis consisting of three vectors and if A is a 5×7 matrix, what is the rank of A?

21. Write $\mathbf{x}$ in terms of the basis vectors.

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$